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Bearing Capacity of Simulated Lunar Surfaces in Vacuum

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The static bearing capacity of a granular material consisting of dry, crushed olivine basalt was determined in air and in a 10^{-6} -mm-Hg vacuum by means of cylindrical probes with a range of diameters Samples with various particle-size distributions (all below 35 mesh) were used for these tests It was found that the packing density of these granular materials was the factor that had the greatest effect on the bearing capacity The minimum bearing capacity of a loosely packed sample with a density of 1 25 g/cm³ was about 0 1 kg/cm² maximum bearing capacity of a densely packed sample with density of 2 1 g/cm³ was about 7 kg/cm² The effects of vacuum were insignificant compared with the effect of packing den-Direct shear tests indicated the cohesion in a few densely packed samples to be 1-2 \times $10^4\,\mathrm{dyne/cm^2}$ For the small probes used, the cohesion was estimated to contribute 85 to 95% of the observed bearing capacity for the densely packed samples but much less for the loosely packed samples

Introduction

THERE is a great deal of design work in progress on the equipment and instrumentation required for the lunar exploration program Precise information with regard to the nature and properties of the lunar surface would be very helpful to those involved in this work Past studies by optical and radio astronomy have indicated that large portions of the moon's surface are composed of a low-density material, probably in powder or porous form 1 It is therefore desirable to determine if properties of granular or porous materials are markedly different in high vacuum from those Studies have been carried out on powdered minerals in vacuum; the properties evaluated include resistance to dynamic penetration, 2 3 thermal conductivity, 4 and land locomotion values 5 In the present work, the static bearing capacity of rock powder was evaluated in air and high vacuum

Materials and Sample Preparation

The simulated lunar surface materials used in this bearing capacity study were prepared from unweathered olivine basalt collected from Pisgah Crater, San Bernardino County,

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Calif A detailed mineralogical description of this volcanic rock is given in Ref 2 The rough breakdown, crushing. pulverizing, and sizing operations were carried out as shown in the flow diagram (Fig 1) Six lots of granular material were obtained; two were well-graded, with nominal mesh sizes of -35 and -150; four were more uniformly sized, with nominal mesh sizes of -35 + 48, -48 + 65, -65 +100, -100 + 150 A particle-size distribution analysis was carried out on each lot; the results are shown in Table 1 and Fig 2

The rock powders were stored in covered containers and were dry except for pickup of moisture from the air Checks made during the program by drying selected samples showed that the water content was always less than 0 1% by weight The density of the solid particles was found to average 2 97 g/cm^3

It was desired to measure the bearing capacity of each lot of powder over a wide range of packing density Data at the lowest densities were of particular interest minimum packing densities, extreme care was taken during the loading of the powders into the sample holder scoop was used for this purpose, and each scoopful was gently placed on the surface of the material previously added Samples with packing densities as low as 1 25 g/cm³ were thus obtained The high-density samples were prepared by filling the container with powder and then tapping it on a solid surface With care in the tapping procedure, spalling of the surface could be avoided, and packing densities as high as 2 29 g/cm³ were obtained

Test Equipment and Procedure

The apparatus used for the bearing capacity measurements was a simple penetrometer (Fig 3) using interchangeable

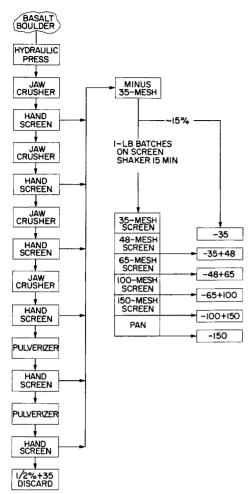


Fig 1 Flow diagram for crushing of basalt (lot 6/61)

circular flat-bottom probes of a range of diameters; such probes are commonly used in soil-mechanics studies. A bushing on the probe stem constrained the probe base to remain horizontal so that soil displacements would take place symmetrically about the probe axis in the absence of boundary effects. Lead-shot loading was used for convenience in adapting the unit for operation in vacuum; with this procedure, the load was measured only at failure. The initial tests were made in air. Multiple measurements were made on every soil sample. Test locations were kept at least two

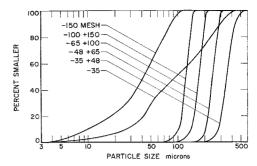


Fig 2 Particle-size distribution of basalt powder

probe diameters away from the container walls and the edges of the craters produced by previous tests; this allowed three to five tests to be made on the 23-cm diam surface of a sample

As might be expected from the known characteristics of soils in air, the load vs depth of penetration behavior (both in air and vacuum) was distinctly different for the loosely packed and the densely packed samples (Fig 4) loosely packed material, the penetration increased gradually as the load increased There was relatively little soil move ment visible except for caving of the walls as the probe penetrated below the surface The mass per unit area at which the depth of penetration was equal to the probe diameter was taken as the mass bearing capacity For densely packed material, radial cracks formed about the probe as the load increased; however, the penetration was negligible until a critical load was reached at which there was sudden vield-The probe would then sink to a depth of about $\frac{1}{2}$ to 1D and produce a cavity with more or less vertical walls The mass per unit area corresponding to the yield load was con sidered to be the mass bearing capacity

The vacuum system consisted of a 45 cm glass bell jar, 15 ft³/min mechanical pump, and a liquid-nitrogen-trapped oil diffusion pump (speed 800 liter/sec at 10⁻⁴ mm Hg). The principal difficulty in determining the bearing capacities of the samples in vacuum was avoiding any disturbance of the packing near the surface during evacuation. Starting at atmospheric pressure, the jar could be evacuated to 10⁻⁶ mm Hg in about 45 min at normal pumping rate. To prevent eruptions or other visible disturbances of the sample, however, the pumping speed was throttled to a point where approximately 6 hr was required to evacuate the space above the sample to a pressure of 10⁻⁶ mm Hg. It should be noted that the subsequent repressurization of a sample could also cause a disturbance of the packing; therefore air had to be

Table 1 Particle-size distribution of basalt powder, batch 6/61^a

		Nominal mesh size					
Measured particle size Mesh Microns		-35 +48, %	-48 +65, %	-65 +100, %	-100 +150, %	-35, %	-150, %
+ 28	>590	0 02	0	0	0	0 13	0
-28 + 35	< 590 > 420	0.94	0 03	0 03	0 02	0 13	0 01
-35 + 48	<420>297	81 96	0 83	0 02	$0\ 02$	8 96	0 02
-48 + 65	<297 > 210	16 00	83 42	0 93	0 02	$12 \ 75$	0 02
-65 + 100	<210 >149	0.45	14 57	79 29	0 63	13 56	0 02
-100 + 150	<149 > 105	$0\ 02$	0 04	$18 \ 25$	85 25	11 99	0 91
-150 + 200	< 105 > 74	0 01	0 03	0 07	$12\ 46$	8 83	15 49
-200 + 270	< 74 > 53	0 01	0 03	0 01	0 08	$9 \ 92$	20 73
-270 + 325	< 53 > 44	0 01	0 02	0 01	0 04	10 80	9 67
	< 44 > 35	0 02	$0\ 02$	0 01	$0 \ 02$	7 98	14 88
	< 35 > 25	0 01	0 01	$0\ 02$	$0 \ 02$	5 43	10 75
	< 25 > 12	0.05	0 10	0 10	0 15	4 57	13 39
	< 12 > 7	0 09	0 24	0 43	0 44	2 99	8 32
	< 7 > 3	0 05	0 21	$0 \ 20$	0 37	1 13	5 69
	<3	0	0	0	0	0	0

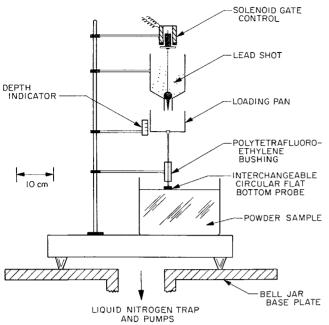


Fig 3 Apparatus for measuring bearing capacity of rock powder Bell jar and vacuum lead-through not shown

admitted slowly after testing The time to change pressure from the 10⁻⁶-mm-Hg range to atmospheric was approximately 6 hr To check packing disturbance during evacu ation and repressurization, bearing capacity measurements were made in air both before and after a test in vacuum

Each sample tested in a vacuum thus had spent 6 hr or more in the vacuum chamber, during which time the pressure in the space above the sample had been reduced from atmospheric to 10⁻⁶ mm Hg Degassing tests on crushed samples of the same basalt have been reported ² These tests, made with a vacuum system of lower pumping speed than that used in the present work, indicate that the vacuum attained was little affected by heating the material As mentioned previously, the rock powders picked up some moisture from the air (less than 01%) To determine whether this had any effect on the bearing capacity as measured, one series of tests was run on material that had been dried in an air oven at 105°C for 24 hr The tests were run in air immediately after the material was removed from the oven

The effects of longer vacuum exposure time were checked by making a measurement on a sample that had been held for 40 hr in a vacuum that reached 2×10^{-6} mm Hg

To examine the effects of probe size, probes with 0 45-, 0 72-, and 1 43-cm radii were used. At least two probe sizes were used for each sample. With dense soil, the load required with the 1 43 cm probe was beyond the capacity of the regular equipment. A few tests were run in air with equipment of greater capacity; this equipment was too large to fit into the vacuum bell jar.

Because of the importance of cohesion in tests with such small probes, cohesion measurements were made in air, on a few samples, with a standard soil-mechanics direct-shear apparatus 6 Both usual normal loads and very low normal loads (giving pressures less than $3 \times 10^4 \, \mathrm{dyne/cm^2}$ on the failure surface) were used The shearing strength (maximum shearing stress sustained) in each test was plotted against the normal stress and a line drawn through these The intercept of this line with the shearing strength axis was taken as the cohesion c; the angle between the line and the normal stress axis was taken as the angle of internal Very small intercepts were found, as reported friction To check the role of friction in the apparatus in contributing to the intercepts, the direct-shear apparatus was set up in the normal way without a sample and operated in the shearing mode with the same vertical load applied to the upper shearing plate as had been used in the low-pressure tests. No movement was discernible on the shear proving ring dial (0 0001 in divisions) as contrasted to maximum readings in the range of 0 5 to 1 9 divisions when the sample tested was present. As a further check, direct-shear tests were also carried out at low normal pressures on a $-20 + 30 \,$ mesh dry uniform Ottawa sand; a zero intercept (no cohesion) was found on the resulting plot of shearing strength vs normal stress

Test Results

The measured bearing capacities in air and vacuum are plotted in Fig 5 against corresponding packing densities for each of the six lots of basalt; the probe sizes used are also indicated

Figure 6 shows, for each nominal mesh size, the range of bearing capacities measured in air and the individual data points measured in vacuum. In the loosely packed condition, the lowest mass bearing capacity measured in vacuum was 0.16 kg/cm² and in air was 0.085 kg/cm². In the densely packed condition, the highest value was 6.2 kg/cm² in vacuum and 7.1 kg/cm² in air. The bearing capacity measurements made in air on the material that had been heated to remove residual moisture were not significantly different and are included in the Fig. 5 air data

Cohesion measurements, on material taken from the surface of samples on which penetration tests had been made, gave intercept values of 1.4×10^4 dyne/cm² for a densely packed sample of -35 mesh material, and 2.1×10^4 dyne/cm² for a densely packed sample of -100 + 150 mesh material

The direct-shear tests on the -35 mesh material gave angles of internal friction of 44° for a densely packed (2 0 g/cm³) sample and 26° for a loosely packed (1 4 g/cm³) sample

The long-time vacuum exposure test on a densely packed —35 mesh sample showed that very much less cracking took place at the sample surface on penetration and that, as the probe was pushed further below the surface, the soil at the edge of the penetration hole fell into an angle of repose without sustaining a vertical wall. This is in contrast to the tests in air and after 6 hr in vacuum, where the probe left a cylindrical hole with vertical walls, and a vertical trench 10 cm deep could be excavated without failure of the vertical

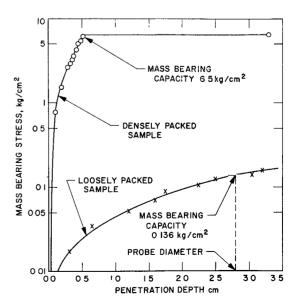


Fig 4 Mass bearing stress vs penetration depth of basalt powder

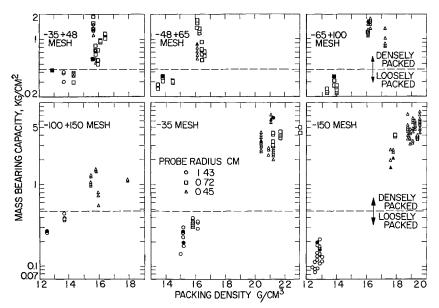


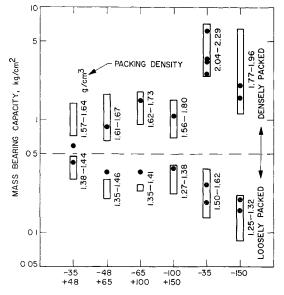
Fig 5 Bearing capacity of basalt powder in vacuum and in air Open symbols designate air tests and solid symbols vacuum tests

wall In a loosely packed sample, attempts to excavate a vertical trench were unsuccessful

Discussion

As the samples used in this study were prepared by hand, there were differences in structure and packing density from sample to sample among those intended to have similar states of packing. Undoubtedly, there were some localized differences on the surface and within each individual sample as well. The differences caused some scatter in the measured bearing capacities; nevertheless, the tests clearly showed that the bearing capacity of the rock powders was the same in a vacuum, of the level and for the times employed, as in air under the test conditions. Figure 6 shows that the vacuum results fall within the range observed in air tests

The factor that had the greatest effect on the bearing capacity was the packing density. The bearing capacities of the crushed basalts varied directly with the packing density. The greatest range of densities and corresponding bearing capacities (Fig. 5) was obtained on the materials with the broad range of particle sizes, i.e., the -35 and -150 mesh



NOMINAL MESH SIZE OF TESTED MATERIAL

Fig 6 Range of bearing capacity of basalt powder for small probes Bar length indicates the range in air Solid circles show individual data points in vacuum

As would be expected from the geometry of particle packing, much higher densities could be obtained with these broad particle-size distributions than with the lots in which most of the particles fell within a narrow range of sizes

In Fig 7 the mass bearing capacities measured for all lots are plotted vs packing density. Averages are shown for air tests and individual data points for vacuum tests. It is clear that most of the results fall within a relatively narrow scatter band, regardless of particle-size distribution or atmospheric pressure, and represent an approximately linear relation between the logarithm of bearing capacity and the packing density. The bearing capacities for the -35 mesh material fall below those for the other particle sizes

The approximate expression for the bearing load P of a circular plate of radius R at a depth D below the surface of a soil of unit weight $\gamma = \rho g$ and cohesion c is q

$$P = \pi R^2 [cN_e + \gamma DN_g + \gamma RN_\gamma] \tag{1}$$

in which, if the units are chosen compatibly, N_c , N_q , and N_{γ} are dimensionless parameters, applying to a circular footing and depending only on the angle of internal friction of the soil

In the tests on the densely packed samples, the depth at which "yielding" took place was relatively small Thus $D \simeq 0$, and Eq. 1) becomes

$$P = \pi R^2 [cN_c + \gamma RN_{\gamma}] = \pi R^2 [cN_c + \rho gRN_{\gamma}] \qquad (2)$$

The maximum bearing capacity p is

$$p = P/\pi R^2 = cN_c + \rho g R N_{\gamma} \tag{3}$$

The gravitational acceleration g will be different on the moon and earth. We may define the mass bearing capacity $p_m = p/g$ and can write Eq. (3) as

$$p = (cN_c/g) + \rho RN_{\gamma} \tag{4}$$

If the results of the penetration tests on densely packed samples are extrapolated to give bearing capacities of larger footings, the contribution due to cohesion remains constant, whereas that arising from the friction term including N_{γ} in Eqs. (3) and (4) increases linearly with the width of the footing. To examine the proportions of the bearing capacity attributable to the different items, Eqs. (3) and (4) may be rewritten in the forms

$$p/\rho gR = (c/\rho gR)N_c + N_{\gamma} \tag{5}$$

or

$$p_m/\rho R = (c/\rho g R) N_c + N_{\gamma} \tag{6}$$

in which the left-hand sides are dimensionless bearing capaci-

ties and $c/\rho gR$ is a dimensionless factor expressing a relation among the cohesion and density of the soil and the size of the footing and gravity. For densely packed samples of the -35 mesh material ($c=1.4\times10^4$ dyne/cm²), $c/\rho gR$ has values of 15 and 9.4 for the small (0.45-cm-radius) and large (0.72-cm-radius) probes, respectively. For densely packed samples of the -100+150 mesh material ($c=2.1\times10^4$ dyne/cm²), the values are 29 and 18

From texts,⁷ theoretical values of N_c and N_{γ} can be obtained as functions of the angle of internal friction of the soil, so that the right side of Eq. (5) and (6) can be plotted vs the angle of internal friction. For a given test, the left side of the equation can be evaluated and from the literature data a value obtained for the angle of friction. In this way it was found that, in a penetration test, the angle of friction of densely packed -35 mesh sample varies from 45° to 47° At 45° the theoretical values of N_c and N_{γ} for a circular footing are 175 and 220, respectively, so that it can be seen from Eq. (5) that the very small cohesion $(1.4 \times 10^4 \, \text{dyne/cm}^2)$ of this material accounts for about 92% of the bearing capacity for the small probe and 88% for the larger probe

Similarly, the angle of friction of the densely packed -100 + 150 mesh sample was found to be about 36°. At this value N and N_{γ} for a circular probe are 65 and 32, respectively, so that the cohesional term provides about 98% of the bearing capacity

All of these calculations were carried out on the basis that the cohesion of the sample at the point of application in the penetration tests was the same as that indicated by the direct shear test. This is inexact, since the cohesion would, in practice, probably vary with the atmospheric humidity and from place to place within the test sample. It can be seen from an examination of the factor $c/\rho gR$ and Eq. (5) that a change as small as $5\times 10^3\,\mathrm{dyne/cm^2}$ in the value of c very markedly affects this factor and thereby the calculated angle of friction and the relative contributions of the N and N_γ terms. Changes of the amount indicated in c would be very hard to measure

In the tests on the loosely packed samples, the bearing capacity was defined as the load per unit area when the probe had penetrated to a depth equal to the diameter of the probe Although the two modes of failure differ, it is convenient to employ Eq. (1) to give

$$p = P/\pi R^2 = cN + \gamma R(2N_q + N_\gamma) \tag{7}$$

Experimental work in soil mechanics has indicated that for a circular footing N_q is approximately $\frac{1}{2}N_{\gamma}$, so that Eq. (7) can be reduced to

$$p = cN_c + 2\rho gRN_{\gamma} \tag{8}$$

or

$$p_m = cN_c/g + 2\rho RN_{\gamma} \tag{9}$$

Since the cohesion in the loosely packed material is considered to be negligible, Eqs. (8) and (9) can be simplified to

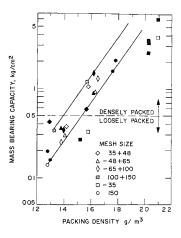
$$p = 2\rho g R N_{\gamma} \tag{10}$$

$$p_m = 2\rho R N_{\gamma} \tag{11}$$

It should be noted from Eqs. (4) and (11) that the bearing capacity should increase with an increase in the radius R or, more generally, with the smaller dimension of the contact area. For the loosely packed material, the bearing capacity is directly proportional to the radius. For the densely packed material, the change would be smaller because of the large contribution attributed to cohesion, which is independent of probe size. On this basis, the range of bearing capacities observed for all samples in vacuum with the small probes would correspond to bearing capacities of 1 3 to 7 0 kg/cm² on a contact area of 10-cm radius

The effect of a difference in gravitational field on the static bearing capacity can be examined by reference to Eqs.

Fig 7 Mass bearing capacity vs packing density of basalt powder for small probes Solid symbols indicate individual data points in vacuum Air test points are averages



(4) and (11) In a loosely packed sample (negligible cohesion), the mass bearing capacity (kilograms per square centimeter) would be the same in any gravitational field, since g does not appear in Eq. (11). The stress supported (dynes per square centimeter or pounds per square foot) would be directly proportional to the gravitational field [Eq. (10)]. In a densely packed sample, the cohesion contributes a major portion to the bearing capacity, and, since in Eq. (4) the cohesion term increases with a decrease in g, the mass bearing capacity (kilograms per square centimeter) would also increase at lower values of gravity

The high bearing capacity of the small probes, the cracks that appeared in the surface of the sample when the probe was pushed into it, the ability of the sample to stand vertically when a trench was excavated in it, and the cohesion intercept values all indicate that in the densely packed state the material used in this investigation possessed cohesion that, though small, was certainly unexpected graded materials are normally considered cohesionless Since the cracking appeared to diminish or disappear for the sample exposed to vacuum for a long time, the presence of molecular layers, possibly of water, on the surface of the grains could account for the cohesion Prolonged heating may remove these surface films, but they probably reestablish before the tests are carried out The significant proportion of fine material in the -35 mesh sample may also contribute to the observed behavior

The inability of the loosely packed samples to stand vertically when an attempt was made to excavate a trench indicated that in this condition the material exhibited negligible cohesion

It would be predicted that effects of atmosphere upon static bearing strength for material of a given internal structure would arise only from the effect of reduced pressure upon the adhesion and the surface friction coefficient of the individual grains and, thereby, on cohesion and the gross friction angle of the material (In dynamic tests, forces associated with movement of air through the pores of the soil also play a role) Since no differences in static bearing behavior in air and vacuum were observed, the surface adhesion and friction of the individual grains was probably not much different as a result of the relatively short exposure to 10⁻⁶ mm Hg from what it was after exposure at 1 atm pressure on the moon's surface is much lower than that used in this series of tests, and the surface of lunar rock particles would be freer of adsorbed gases This would tend to raise the friction and adhesion between particles and, as a conse quence, the bearing capacity It is also possible that some sintering of the lunar dust occurs over the years

In applying the results of the present work, perhaps the most important finding is that vacuum, at least to the levels studied, appears to have little effect on soil bearing capacity

and that the bearing capacity of a given soil in vacuum is certainly at least as high as in air (The effect of a small amount of cohesion on the bearing capacity as measured by small probes is also of interest) The state of packing of the soil is the major factor in determining its mechanical properties It is extremely difficult to maintain very low packing densities on dry rock powders under terrestrial conditions of mineral friction and adhesion Even the slightest vibration or other disturbance induces an increase in packing density and therefore a major increase in bearing capacity. The lunar surface is subject to meteoroid bombardment, severe thermal fluctuations, and possibly some seismic disturbances It seems most unlikely, then, that rock powders comprising contacts between the particles without adhesion could remain at extremely low densities corresponding to degrees of packing as low as or lower than some employed in the present work If adhesion is present, it is possible that very lowdensity granular materials could exist on the lunar surface in spite of vibrational disturbances: the adhesion would increase their strength

Conclusions

On the basis of the work described, the following conclusions were reached:

- 1) The static load bearing capacity of rock powder is the same after a short period in a vacuum of 10^{-6} mm Hg as it is in air
- 2) The bearing capacity is dependent principally on the state of packing of the material
- 3) In vacuum the lowest mass bearing capacity, measured with a probe of radius 1 43 cm, was 0 17 kg/cm² for a very loosely packed sample For a very densely packed sample, the highest mass bearing capacity was about 7 kg/cm²;

because of the small probe size (0 45-cm radius), this capacity arose primarily from a small amount of cohesion The corresponding bearing capacities on a contact area of 10-cm radius, as calculated from these measurements, would be approximately 1 3 and 7 0 kg/cm 2

- 4) Small amounts of cohesion may make an important contribution to soil bearing capacities measured with small probes
- 5) Analysis indicates that for a given loosely packed sample the mass per unit area which can be sustained should be the same in the moon's gravitational field as in that of the earth. The stress that can be sustained should be one-sixth as great. When cohesion is present in a densely packed sample, the mass per unit area which can be sustained should be greater in the moon's gravitational field than on the earth

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